

Dialogue Games as a Semantics

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Introduction

Dialogical Logic

- Logical framework based on games
- Developed within the constructivist school of Erlangen
- A Proponent (**P**) and an Opponent (**O**) play games with logical formulas according to certain rules
- The existence of winning strategies for **P** in such games corresponds to notions like truth and validity
- Title of an important paper by Kuno Lorenz (1968):
“Dialogspiele als semantische Grundlage von Logikkalkülen”

Aims of this talk

Part I:

- General philosophical/conceptual remarks concerning Dialogical Logic
- More specifically: Dialogues as providing a **semantic** foundation (How does the **conception of meaning** in the dialogical framework look like)

Part II:

- Some remarks on Giles's Game

Semantic Approaches

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graph TD; A[Semantic Approaches] --> B[Denotational/referential approaches]; A --> C[Use-based approaches];
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Denotational/referential
approaches
(f.e. model theory)

A broadly
Fregean/Wittgensteinian(I)
picture of language
and meaning

Use-based
approaches

A broadly
Wittgensteinian(II)
picture of language
and meaning

Use-based semantic approaches

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graph TD; A[Use-based semantic approaches] --> B[Proof-theoretic approaches  
(f.e. Natural Deduction)]; A --> C[Game-theoretic approaches  
(f.e. Dialogical Logic)];
```

Proof-theoretic
approaches
(f.e. Natural Deduction)

Rules how to use
expressions in proofs

Game-theoretic
approaches
(f.e. **Dialogical Logic**)

Rules how to use
expressions in
language games

How many (epistemic) subjects are involved?

Model-theoretic approaches: 0

(\Rightarrow an „anatomic“ conception of meaning)

Proof-theoretic approaches: 1

(\Rightarrow a „solipsistic“ conception of meaning)

Game-theoretic approaches: 2 (or more)

(\Rightarrow an interactive/communicative conception of meaning)

The rules of Dialogue Games

Particle rules

(They determine how formulas, containing the respective particles, can be attacked and defended)

Structural rules

(They determine the general course of the game)

Claim

Those game rules which constitute a semantics
have to be **player-independent (player-
symmetric)**

The Particle Rules (I)

	Attack	Defence
$\neg\alpha$	α	\otimes
$\alpha\wedge\beta$?L(eft) ----- ?R(ight)	α ---- β
$\alpha\vee\beta$?	α ---- β
$\alpha\rightarrow\beta$	α	β

The Particle Rules (II)

	Attack	Defence
$\forall\rho\alpha$	$?_c$ (The attacker chooses)	$\alpha [c/\rho]$
$\exists\rho\alpha$	$?$	$\alpha [c/\rho]$ (The defender chooses)

Observation:

The particle rules **are** player-independent!

Aren't there player-dependent (player-asymmetric) structural rules???

- 1) What about the starting rule?
- 2) What about rules like Felscher's rule (E)
“**O** can react only to the immediately preceding move by **P**. No such restriction applies to **P**.”
- 3) What about the formal rule?
“**O** may attack any atomic assertions of **P**. **P** is only allowed to attack an atomic assertion of **O** if has already attacked the same atomic assertion by **P** before.”

Answers

- 1) The starting rule is not really player-dependent (player-asymmetric).
- 2) Rules like Felscher's (E) are player-dependent (player-asymmetric), but they shouldn't be part of a dialogical system which is supposed to provide a semantics. (Though it might be OK to introduce them for certain purposes and motivate them strategically).
- 3) The formal rule is player-dependent (player-asymmetric) and it is part of the dialogical framework, but it doesn't belong to the meaning-constituting rules/it doesn't belong to the semantic part of the framework. (More on the formal rule later!)

Meaning in Dialogical Logic

Particle rules

- ⇒ provide the meaning of the logical particles
(local meaning)
how to attack and how to defend

Particle rules + structural rules (without the formal rule) + X
(meanings of the atoms)

- ⇒ Provide the meaning of propositions
(global meaning)
how to play dialogical games

Plays vs. Strategies

Level of **plays**

⇒ Game rules
(**how to play; meaning** is constituted)

Level of **strategies**

⇒ Strategic rules
(**how to play well**; requires more than playing games, requires studying games; existence of winning strategies; **notions like truth and validity** can be defined)

Back to the particle rules

	Attack	Defence
$\neg\alpha$	α	\otimes
$\alpha\wedge\beta$?L(eft)	α
	-----	----
	?R(ight)	β
...		

Observation: Attacks and defences are always less complex than the attacked formula

\Rightarrow Plays unavoidably reach the atomic level

Question

What happens at the atomic level?

Digression: Hintikka's GTS and the atomic level

In GTS the games are always played **given a certain model** (and the players know about the model!): Atomic formulas are evaluated according to the model and the result of a play can be accordingly determined.

GTS:

- Game-theoretic semantics for the logical connectives
- Model-theoretic semantics for the atoms

⇒ GTS is a combination of a game-theoretic and a model-theoretic approach!

Validity in Hintikka's GTS

Validity in GTS:

For every model there is a winning strategy (for the first player)
(validity as general truth/existence of a winning strategy)

Question

What's the point of game-theoretic approaches in logic? Isn't all this just a reformulation of well-known things using games talk?

Answer: Yes, indeed. So far...

But: The games approach opens up new possibilities, especially the transition to **games with imperfect or incomplete information**

Hintikka's IF Logic

Main idea:

When concerned with formulas with nested quantifiers, a player having to choose how to attack or defend a quantifier, might lack information about how the other player attacked or defended another quantifier earlier on. In this sense the second quantifier is independent from the first.

Hintikka's IF Logic: Examples

Slash notation: $\forall x(\exists y/\forall x) R(x,y)$

(Only a uniform strategy for choosing y is possible.)

Consequently: $\forall x(\exists y/\forall x) R(x,y)$ and $\exists y\forall x R(x,y)$ have the same truth-conditions.

But: The expressive power of IF logic exceeds that of first-order logic.

For example: $\forall x\exists y\forall z(\exists w/\forall x) R(x,y,z,w)$

Back to Dialogical Logic and back to the formal rule

Claim:

The formal rule is motivated by strategic considerations.

The deeper motivation of this rule can best be explained with a transition to **games with incomplete information**:

Suppose that **P lacks information about the atomic level**. Let's say that there are rules about how to attack and defend atomic formulas, but **P** doesn't know how they look like. Thus, he also doesn't know which atomic formulas yield a win or a loss.

P lacks information. Is he lost?

Two cases:

- 1) **O** states an atomic formula; **P** is unable to attack as he lacks information about how such an attack looks like
- 2) **P** states an atomic formula; **O** attacks it and **P** is unable to react as he lacks information about how a defense looks like

Question:

Is it nevertheless possible for **P** to have a winning strategy?

Copycat does the trick!

Answer:

Yes! Because of a **copycat strategy**.

If **O** has already stated an atomic formula before, **P** is safe when stating this atomic formula himself as **O** can't successfully attack because he then indirectly attacks himself. (If **O** attacks, **P** can copy this attack, and if **O** then defends against the attack, **P** can copy the defense etc etc.) So, in this situation **P** can never loose.

This idea is captured by the **formal rule**.

Validity in Dialogical Logic

The standard conception (validity as general truth):

Validity as truth in every model

Or: Validity as the existence of a winning strategy given any model (see GTS)

The dialogical conception (validity as formal truth):

Validity as the existence of a winning strategy despite lacking information about the atomic level

Or: Validity as the existence of a winning strategy when the formal rule is in effect

The role of the formal rule

Making the plays independent of the meaning of the atoms

⇒ Transition to logic!

Part II: Giles's Game

A brief description of the essentials (cf. Fermüller, “Revisiting Giles's Game”):

- Complex propositions are decomposed according to the standard propositional rules
- Every play ends in what is called an elementary state $[p_1, \dots, p_m \parallel q_1, \dots, q_n]$, p_1, \dots, p_m being the atomic propositions **O** has committed to, and q_1, \dots, q_n being the atomic propositions **P** has committed to.
- **P** wins a play according to an assignment of risk values $\langle \cdot \rangle^r$ iff $\langle p_1, \dots, p_m \rangle^r \geq \langle q_1, \dots, q_n \rangle^r$
- Giles's Game is a game-theoretic characterization of Łukasiewicz Logic \mathcal{L} :
 α is valid in \mathcal{L} iff for all risk value assignments there exists a winning strategy for **P**.

First observation about Giles's Game

Although it is most of the time said that Giles's Game is based on Lorenzen style dialogues, it is clearly much closer to Hintikka's GTS games than to Dialogical Logic:

- It is a combination of a game semantics for the logical connectives plus assignments of values to the atoms. (Actually, when just allowing for risk values 0 and 1 we essentially get GTS games for classical logic.)
- The conception of validity clearly is a standard one: validity as general truth (existence of a winning strategy under all risk value assignments).

Question 1

Why does Giles use a games formulation at all?

Why not just assign truth-degrees/probabilities/risk values to the atoms and determine how the truth-degrees/probabilities/risk values of complex formulas depend on those of the atoms?

A putative answer: Giles is interested in a logic for a physical language about elementary experiments with dispersion. And he thinks that „probability assignments are subjective“ (Giles 1977, p. 26)

So he needs subjects (= the players)!

Question 2

Why Hintikka's GTS way, and not the dialogical way?

Claim:

Formulating the game in the dialogical way would have been much more in accordance with Giles's underlying motivation and background assumptions!

Giles on „tangible meaning“

“[A] *tangible meaning* for an assertion consists of an exact description of some *obligation* undertaken by the assertor.” (Giles 1977, p. 27)

If we replace „obligation“ by „tacit commitment“ and skip the „some“, this sounds very much like an attractive account of what is going on in the particle rules of dialogical logic:

The particle rules make explicit the tacit commitments when asserting complex proposition.

Proposal for reformulating Giles's Game

Instead of assigning risk values to atomic propositions at the end of a play, we add subjective (player-dependent) probabilities/“assertion certainties“ to the moves of the players in the game itself:

A move might then look like:

$\vdash_{0,5} \alpha$ (means: α is asserted with subjective probability $\geq 0,5$)

Particle rules then might look like this:

	attack	defence
$\vdash_n \alpha \wedge \beta$?L(eft)	$\vdash_n \alpha$
	-----	----
	?R(ight)	$\vdash_n \beta$

Conjecture

Add to this a formal rule like the following one:

P is allowed to make a move $\vdash_n \alpha$ (α being an atom) iff **O** has made a move $\vdash_m \alpha$ with $m \geq n$ before.

By adequate formulations of the rest of the game rules one should get a dialogical system for \mathbb{L} (and by slightly changing the rules, one for other systems of fuzzy-logic, too).

(The idea of using indexed turnstiles is taken from Rückert, „Logiques dialogiques ,multi-valents“)

Final remark (not too serious)

„For instance, he might say it [„the probability of E exceeds $\frac{1}{2}$ “] means that the limit of the ratio of „yes“ outcomes to „no“ outcomes in an infinite series of trials would exceed $\frac{1}{2}$.“ (Giles 1977, p. 27)

I don't dare to hope that my talk didn't contain any mistakes at all, but I really hope that I didn't make a mistake as blatant as this one by Giles. 😊

THANK YOU!